การลอกเลียนผลตอบแทนของอ๊อปชั่นและกลยุทธ์การซื้อขายแบบแสวงหากำไร โดยศึกษาผลกระทบด้านสภาพคล่องและต้นทุนของการซื้อขาย

Option Replication and Arbitrage Trading Strategy with Liquidity and Transaction Costs

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วิทยานิพนธ์ฉบับนี้ ทำการศึกษาราคาของออปชั่นโดยการใช้กลยุทธ์แบบออฟติมอล ซึ่งเป็นการสร้างพอร์ตโฟลิโอเพื่อเลียนแบบมูลค่าของออปชั่นโดยที่สินทรัพย์อ้างอิงมีความเสี่ยงด้านสภาพคล่องและต้นทุนการซื้อขาย กลยุทธ์นี้มีข้อดี คือ สามารถยืนยันได้ว่า มูลค่าของพอร์ตโฟลิโอนั้นจะไม่น้อยกว่าผลตอบแทนของออปชั่น ณ วันสุดท้าย โดยในการศึกษาใช้ข้อมูลราคาออปชั่นจากตลาดอนุพันธ์แห่งประเทศไทย เนื่องจากการซื้อขายแบบต่อเนื่องไม่สามารถทำได้จริงในทางปฏิบัติดังนั้น ในการศึกษาจึงใช้กลยุทธ์การซื้อขายแบบแยกช่วงเวลา จากผลการศึกษายังพบว่า ตัวแปรด้านสภาพคล่องนั้นมีค่าเป็นบวกและมีนัยสำคัญทางสิถิติ ซึ่งสามารถตีความได้ว่า เส้นอุปทานนั้นมีลักษณะลาดเอียงขึ้นจากซ้ายไปขวาโดยสอดคล้องกับทฤษฎีด้านเศรษฐศาสตร์จุลภาคที่ว่า ราคาของสินทรัพย์อ้างอิงขึ้นอยู่กับประเภทของคำสั่งซื้อขายและขนาดของการซื้อขาย นอกจากนั้น ความแตกต่างของราคา ระหว่างราคาที่ได้จากการใช้แบบจำลองที่คำนึงถึงความเสี่ยงด้านสภาพคล่องและต้นทุนการซื้อขาย กับราคาตลาดของออปชั่นในแต่ละสัญญา สามารถใช้แสวงหากำไรได้เมื่อราคาจากแบบจำลองต่ำกว่าราคาตลาด

คำสำคัญ : การตั้งราคาออปชั่น การสร้างพอร์ตโฟลิโอเลียนแบบมูลค่าออปชั่น กลยุทธ์การซื้อขาย ความเสี่ยงสภาพคล่อง ต้นทุนการซื้อขาย

ABSTRACT

This study investigates the pricing of option by using the optimal hedging strategy for super-replicating an option in which the underlying asset is not perfectly liquid and having some transaction costs. This optimal hedging strategy provides an advantage of assuring that the value of the replicating portfolio is not less than the option's liability at maturity. The theoretical analysis and empirical evidence are conducted in the Thai option market. Since the continuous trading is impossible to implement in practice, the discrete trading strategies are defined in this study. The empirical results reveal the positivity significant of liquidity parameter which implied the upward-sloping of supply curve. This is consistent with the market microstructure literature in which purchases executed at higher prices and sales at lower prices. Moreover, the different between model prices (with liquidity and transaction costs included) and actual market prices in each option series will offer the trading strategy to do arbitrage when the model prices are lower than the market prices.

Keywords : pricing option, portfolio replication, trading strategy, liquidity cost, transaction cost

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INTRODUCTION

The standard models like the Black-Scholes-Merton option pricing formula assume perfect frictionless markets. The replicating portfolio consists of a long position in risky assets and short position in bonds. This portfolio will try to minimize the initial cost of obtaining a terminal payoff that is at least as large as that from the option being hedged which can be referred as super-replication of the required cash flow. The weights of this portfolio are rebalanced in each period so that it replicates the payoff of the option contract at maturity. Under this assumption, rebalancing is costless. But when there are the liquidity and transaction costs, this is no longer the case.

Cetin, Jarrow, Protter and Warachka (2005) define the liquidity risk as the increase of variability in realized returns from forming a replicating portfolio or implementing a hedging strategy because of the price impact of random transactions. In particular, the corresponding price impacts for the series of transactions required to hedge an option are stochastic since they depend on the evolution of the stock price.

On the other hand, the transaction cost is the trading cost that is charged on the change in the net stock position. Edirisinghe, Naik and Uppal (1993) show that with transactions costs, the optimal trading strategy depends on the entire history of the stock price. Consequently, the constraints in the optimization have to be imposed path-by-path. Moreover, Leland (1985) states that the transactions costs themselves are random and will add significantly to the error of the Black-Scholes replicating strategy.

The purpose of this study is to examine the pricing of derivatives using discrete trading strategies when the underlying asset is not assumed to be perfectly liquid and have some transaction costs. Specifically, we study the pricing and hedging of a European call option on a stock in the Thai market by applying the optimal hedging strategy of portfolio replication. In the portfolio replication of a given payoff, the dynamic programming will be employed based on a tree diagram. A backward recursion is used for solving the minimization of the portfolio value and finding out the liquidity and transaction costs.

Our hypothesis in this study is that supply curve liquidity is upward sloping. This is consistent with the market microstructure and the economics literature. Specifically, we incorporates the size (number of shares) and direction (buy versus sell) of a transaction to determine the price at which the trade is executed. The greater an asset’s liquidity, the more horizontal its supply curve.

This study of options pricing with both liquidity and transaction costs will contribute to the new empirical results. Moreover, the liquidity and transaction costs factors are expected to have a significant impact on the option valuation.

METHODOLOGY

To value a European call option, we assume the stock’s supply curve satisfies

$$S(t, x) = e^{\alpha x} S(t, 0) \text{ with } \alpha > 0 \qquad (1)$$

where

$$S(t,0) \equiv \frac{s_t}{e^{rt}} = \frac{s_0 e^{\mu t + \sigma w_t}}{e^{rt}} \qquad (2)$$

for constants μ and σ, with W_t denoting a standard Brownian motion.

Denoted that S_t represents a stock price at time t and S(t, 0) represents the transaction price, per share, at time t □ [0,T] that a trader pays/receives for order flow x normalized by the value of a money market account. While S(t, x) is the marginal stock price which a positive order (x > 0) represents a buy, a negative order (x < 0) represents a sale and x=0 corresponds to the marginal trade. Moreover, α is defined as the liquidity cost parameter and r is denoted as the spot rate of interest. It is important to emphasize that the supply curve given in expression (1) is stochastic. After a trade is executed, a new supply curve S(t, x) is generated for subsequent trades.

Liquidity costs estimation

A simple regression methodology is employed to estimate the liquidity parameter α. Let τ_i denote the time index with corresponding order flow x_{τ_i} and stock price S(τ_i, x_{τ_i}) for every transaction i = 1, . . . , N in a given day. Thus, we are led to the following regression specification

$$\ln\left(\frac{s\left(\tau_{i+1}, x_{\tau_{i+1}}\right)}{s\left(\tau_i, x_{\tau_i}\right)}\right) = \alpha\left(x_{\tau_{i+1}} - x_{\tau_i}\right) + \mu\left(\tau_{i+1} - \tau_i\right) + \sigma\varepsilon_{\tau_{i+1},\tau_i} \qquad (3)$$

The error $\varepsilon_{\tau_{i+1},\tau_i}$ equals $\varepsilon\sqrt{\tau_{i+1} - \tau_i}$ with ε being distributed N(0, 1). Observe that the left side of equation is the percentage return between two consecutive trades. For any discrete trading strategy, the liquidity cost equals

$$L_T = \sum_{j=0}^{N} \left[x_{\tau_{j+1}} - x_{\tau_j}\right] \times \left[S\left(\tau_j, x_{\tau_{j+1}} - x_{\tau_j}\right) - S\left(\tau_j, 0\right)\right] \qquad (4)$$

In this study, we assume $\alpha(\tau_j)$ and S(τ_j , 0) terms are fixed at their initial value α and S(0, 0). In particular, we define the approximate liquidity cost ($\overline{L}_T$) as

$$\overline{L}_T = \sum_{j=0}^{N} \alpha\left(\tau_j\right) S\left(\tau_j, 0\right)\left[x_{\tau_{j+1}} - x_{\tau_j}\right]^2 \equiv \alpha S(0,0) \sum_{j=0}^{N}\left[x_{\tau_{j+1}} - x_{\tau_j}\right]^2 \qquad (5)$$

Transaction costs estimation

The trading cost will be charged on the change in the net stock position, whether buying or selling stocks. The transaction cost can be divided into two types: variable costs and fixed cost.

$$R_T = \sum_{j=0}^{N}\left[\left|x_{\tau_{j+1}} - x_{\tau_j}\right| \theta S(t,x) + \phi I(t,x)\right] \qquad (6)$$

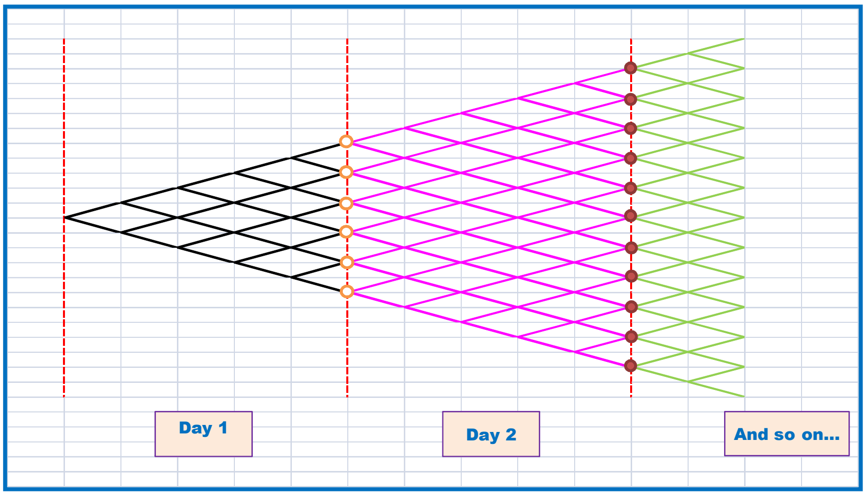
Let □ denote a fixed component to the trading costs and θ is the variable component. Where I(t, j) = 0 if $x_{t_{j+1}} = x_{t_j}$, and 1 otherwise. R_T is the total transaction costs and N is the number of periods considered.

Supposed an investor wishes to replicate a claim maturing at T. An investor who buys one share of stock when the stock price is S(t, x), pays S(t, x)(1+θ). On the other hand, when establishing a short position, the investor receives S(t, x)(1-θ).

The multi-period of binomial tree construction

The following offers a brief summary of the steps required to implement the dynamic programming procedure using a binomial stock price process. Consider a recombining binomial tree with an initial stock price at time 0 denoted by S, where U and D is the up and down factors. This study limits the price of stock by introduced the maximum change of the price by 1.5% in one day. Thus the up and down factors will be determined as $(1 + 0.015)^{1/N}$ and $(1 - 0.015)^{1/N}$, respectively. Which N is defined as the frequency of price change per day. In this study, the binomial tree will be constructed by more than two possible events in one day. We determine that the stock price can go up or down 5 times in one day (N=5). This diagram can be shown in Figure 1.

Figure 1: The binomial stock price process with N=5



The tree has many possible closed prices at each day which makes it more flexible and consistent with the real price changes, so the trading strategy will be chosen more accurately once the closed price is known.

Optimal discrete option hedging strategies

To investigate the pricing of derivatives using discrete trading strategies when the underlying security has liquid and have some transaction costs, the optimal hedging strategies for super-replicating an option will be employed because it is often invoked in the incomplete markets literature due to its independence from investor preferences and probability beliefs. The dynamic programming will be used to construct a tree diagram by implementing a backward recursion to solve for the minimization of the portfolio value and find out the liquidity and transaction costs. The portfolio value (Z) equals the amount of money in bank account plus the value of stock holding. This value can be called as the cost of obtaining a terminal payoff that is at least as large as that from the option being hedged which can be referred as super-replication of the required cash flow.

Let $Z_t = X_tS(t, 0) + Y_t$ denote the time t marked-to-market value of the replicating portfolio where X_t represents the trader's aggregate holding of stock at time t and Y_t is the aggregate position in the money market account at time t. For super-replicating a call option, the optimization problem is:

$$\min_{(x,y)} Z_0 s.t. Z_T \geq C_T = \max\{S(T,0) - Ke^{-rT}, 0\} \qquad (7)$$

Subject to

$$Z_T = y_0 + x_0 S(0,0) + \sum_{j=0}^{N-1} x_{\tau_{j+1}} [S(\tau_{j+1},0) - S(\tau_j,0)] - L_T - R_T \qquad (8)$$

At an intermediate time t $\geq$ 0, this problem is written as

$$\min_{(x,y)} Z_t s.t. Z_T \geq C_T = \max\{S(T,0) - Ke^{-rT}, 0\} \qquad (9)$$

Denoted by x_0 the initial stock holding at the beginning of trade and y_0 represents initial position in the money market account. S(0,0) represents an initial stock price in which C_T denotes as the options payoff with a strike price K and maturity T.

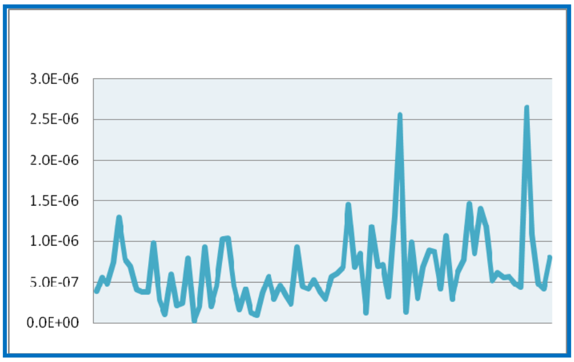
EMPIRICAL RESULT AND ANALYSIS

To investigate the option pricing with liquidity and transaction costs of trading underlying SET 50 index (via TDEX), we consider three series of at-the-money call options maturing at December 2008, March 2009 and June 2009. The results of the study are given below.

Liquidity parameter

To estimate the liquidity parameter (α), a simple regression methodology is employed by using the equation (3).

Figure2: Daily Estimated α



A series of TDEX data including prices, trading volumes and time of trading transactions is used for daily estimation of α and μ. We use August 2008 to November 2008 for the sample period with a total of 80 trading days. The estimated daily α's are positively statistically significant at the 5% level for 74 days out of 80 days while μ's are significantly different from 0 for only 48 days. The positivity of α assures that the supply curve is upward sloping.

The plot of daily estimated α's during the sample period is shown in the Figure 2. The average of these α's equals 6.49092×10^{-7} which will be used as the liquidity parameter in the dynamic programming.

The average of estimated α's means that if the net stock position is changed by 100 shares, the percentage return will change by 6.49092×10^{-5} when holding other parameters constant.

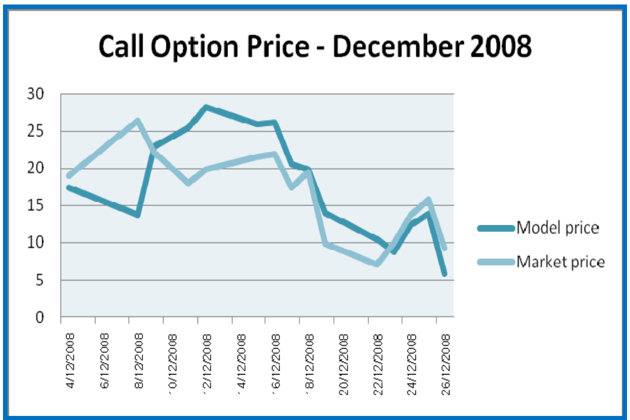
Transaction cost parameter

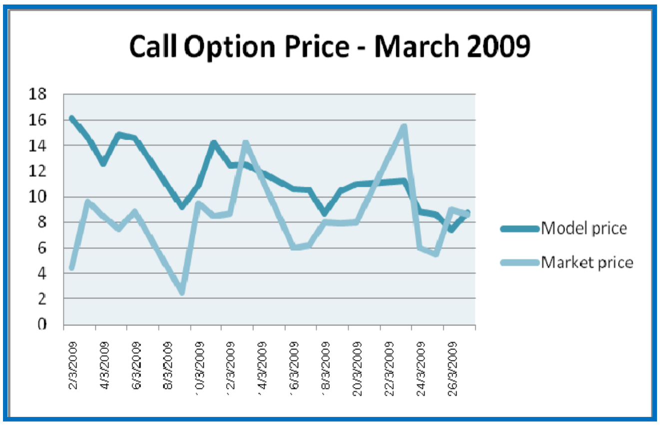
The transaction costs of internet trading are defined by the stock exchange of Thailand which are composed of two components. The first component is the commission fee given by (0.01%)*(number of shares traded)*(stock price per share). The second component is 7% tax of the commission fee. Thus transaction costs parameter (θ) equals 0.00107. This means that if we buy more stock by 100 shares, we have to pay 0.107*(stock price per share) for this trading transaction.

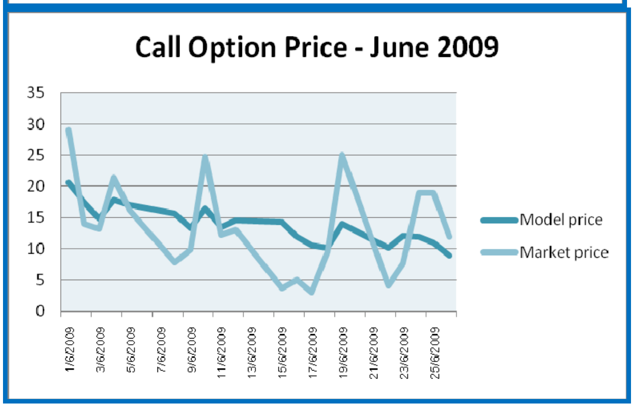
Call option price

The call option prices from our model (with liquidity and transaction costs included) and the actual market option prices are shown in the figure 2 below. These options are at-the-money call options maturing in December 2008, March 2009 and June 2009.

Figure 3: Option Prices Comparing







Descriptive Stat of Pricing Error			
	Dec-08	Mar-09	Jun-09
Mean	-0.98708649	-3.26936232	-0.33919756
Variance	26.583004	12.8733008	40.0346257
Standard deviation	5.15587083	3.58793824	6.32729213

The results show that there are the prices different between the model prices and market prices. The market prices of all option series are usually higher than the model prices when reaching to the maturity. As we have seen in the graph, the model prices are less fluctuated than the market prices in March 2009 and June 2009 series.

Statistics of pricing errors of each option series are shown in the descriptive stat table above. On average, there is a negative pricing error in all series. This means that the market prices the options lower than the model prices on average. In March 2009 series, the mean of pricing error is highest compared to the other series. The variance and standard deviation of June 2009 series is the highest because the market prices in this month are highly fluctuated. Whereas the model prices are quite smoother, this brings about the high statistical in standard deviation and variance of the pricing error in this month.

In order to price the option, this study has developed the pricing model that takes into account the liquidity and transaction costs in the replicating portfolio. Due to market imperfection, we expect that the liquidity cost will have a significant impact on prices and hypothesize the existence of a stochastic supply curve. The empirical result shows that the liquidity parameter is positively statistically significant at 5% level which confirms the upward-sloping of the supply curve. This is consistent with the market microstructure literature in which purchases executed at higher prices and sales at lower prices. However, the estimated liquidity cost is very small and not consistent with our expectation. Due to the small value of liquidity cost, we can infer that the underlying security market in Thailand is liquid while the value of transaction cost depends on the amount of traded. The higher change in the net stock position, the more expenses you have to pay for the transaction cost.

The empirical results demonstrate that, in each call option series, the market prices are deviated from the model prices. There are no exactly patterns in the prices different. In some periods, market prices are higher than the model prices and have lower prices in other periods. The pricing error is defined as the market price minus the model price. If it is positive, it means that the market prices have higher prices than the model prices (shown as the market prices line is above the model line in the Firgure3). On the other hand, if an error is negative, the market prices have lower prices than the model prices. The pricing error of the call option is quite large because the market prices have highly fluctuation.

The different in call option prices between the model and the actual market data offer the arbitrage trading strategies. If the option market price is higher than the model price, we will sell the option and then forming a portfolio which replicating the option’s payoff. On the other hand, the trading strategy will be changed if the option market price is lower than the model price. That is the trading strategy will be long the option and short stock. The stock will be short as much as, at the end, the payoff of option can covering all the expenses of buying the stock back. Hence, these trading strategies can be used to examine the arbitrage opportunity.

CONCLUSION

This study represents the option pricing with liquidity and transaction costs in the underlying security market. The theoretical analysis is conducted, along with the empirical results from the Thai option market.

The optimal hedging strategy for super-replicating an option are introduced to price the option in which the liquidity cost is incorporated into the model in the form of the stochastic supply curve. The supply curve is constructed in such a way that the underlying security prices depend on order direction and volume.

Specifically, purchases are executed at higher prices while sales are executed at lower prices. In addition, the larger the order size, the higher the price deviation from the marginal stock price.

By using the dynamic programming, the multi-period binomial tree diagram is constructed. To find out the minimization of the portfolio value and the liquidity and transaction costs, a backward recursion is employed to work out.

The results show the important evidence in option prices and the liquidity and transaction costs factors. Firstly, the liquidity parameter has positively statistically significant at 5%. This positivity assures the upward-sloping of supply curve and provides the consistency results with the market microstructure literatures. Secondly, the estimated liquidity parameter is very small while the transaction cost depends on the stock price and the number of stock traded. Thirdly, the pricing errors of call option in all three series are quite high because the market prices are very volatile. Finally, due to the different in model prices and market prices, there are two trading strategies; (1) selling the option and buying the stock and (2) selling the stock and buying the option. These trading strategies can be used to investigate an arbitrage opportunity in Thai market for further study.

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